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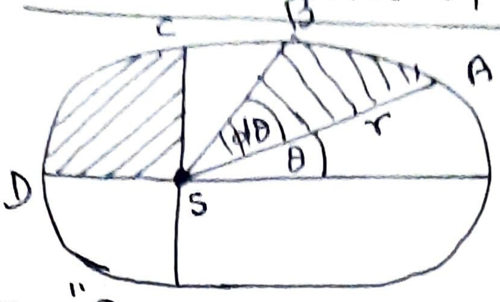
KEPLER'S LAW OF PLANETARY MOTION



Ist law:- "Each planet moves in an ellipse with sun at its focus."

The ecentricity of the orbit of a particle moving under attractive inverse square law force is given by $e^2 = 1 + \frac{2EJ^2}{mc^2}$ where J is angular momentum of the particle of reduced mass m, Total energy E

$$\therefore E = -\frac{mc^2}{2J^2}(1-e^2) \quad \text{---(1)}$$

In case of planet revolving around sun if $e<1$, the total energy of the system is negative. The planet remains bound to the attracting centre, the sun. planet is bounded by sun can not escape from it, it moves around the sun in closed elliptic orbit.

2nd law:- Planet sweeps out equal areas in equal interval of time. i.e. Radius vector drawn from sun to planet sweeps out equal areas in equal interval of time.

Let dA = Area of $\Delta SAB = \frac{1}{2} \times (SA) \times AB = \frac{1}{2} \times r \times r\,d\theta$

$$\Rightarrow dA = \frac{r^2 d\theta}{2}. \quad \therefore \frac{dA}{dt} = \frac{\frac{1}{2} r^2 d\theta}{dt} = \frac{1}{2} r^2 \left(\frac{d\theta}{dt}\right)$$

But $\frac{d\theta}{dt} = \omega$ $\quad \therefore \frac{dA}{dt} = \frac{1}{2} r^2 \omega$ ---(i).

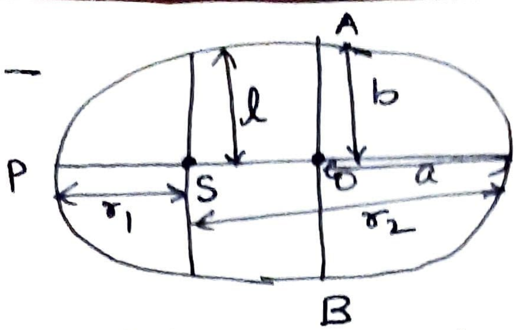
If angular momentum is L and mass of planet is m

$$\therefore L = I\omega = (mr^2)\omega \quad \text{---(ii)}$$

$$\therefore L = mr^2\omega \qquad \therefore \frac{dA}{dt} = \frac{mr^2\omega}{2m} = \frac{L}{2m} \quad \text{(iii)}$$

$$\therefore \boxed{\frac{dA}{dt} = \text{Constant}}$$

3rd Law :-



Square of period of revolution of planet about the Sun divided by cube of majour axis is constant.

Let a and b are semi majour and minor axis

$\therefore$ Area of majour circle $= \pi ab$

If Time period of planet $= T$

$$\therefore T = \frac{\pi ab}{L/2m} = \frac{2m\pi ab}{L}$$

$$\Rightarrow T^2 = \frac{4\pi^2 m^2 a^2 b^2}{L^2}$$

$\therefore$ Here $L = \frac{b^2}{a}$ $\therefore b^2 = La.$

$$\therefore T^2 = \frac{4\pi^2 m^2 a^2 (la)}{L^2}$$

$$= \left(\frac{4\pi^2 m^2 l}{L^2}\right) a^3$$

$\therefore T^2 \propto a^3 \Rightarrow \boxed{\frac{T^2}{a^3} = \text{constant}}$