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QUANTUM PHYSICS (Part II)

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SCHRODINGER TIME INDEPENDENT EQUATION:-

The wave function ψ Contains all the information regarding a quantum mechanical equation. In such a system, the total probability does not change with time. Although it contains states where the probability density at each point remains independent of time and in consequence of this expectation values of all dynamical variables are also constant in time. Such states are called stationary states and the wave function ψ the stationary state solution. Such states also posseses a definite value of energy. The corresponding wave equation is called the stationary state schrodinger wave eqn. or time independent schrodinger wave eqn..

Let us Consider

$$\psi(r, t) = \psi(r) f(t) \quad \text{--- (1)}$$

$$\Rightarrow i\hbar \psi \frac{df}{dt} = -\frac{\hbar^2}{2m} f \nabla^2 \psi + V f \psi$$

Dividing Throughout by ψf.

$$\frac{1}{f} i\hbar \frac{df}{dt} = -\frac{\hbar^2}{2m} \frac{1}{\psi} \nabla^2 \psi + V(\vec{r}) \quad \text{--- (2).}$$

The eqn for f is

$$i\hbar \frac{df}{dt} = Ef \quad \text{--- (3).}$$

and eqn for ψ is

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})\right] \psi = E\psi \quad \text{--- (4)}$$

$$\text{or } H\psi = E\psi \quad \text{--- (4')}$$

Where H is operator (Hamiltonian)

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})$$

Corresponding to the total energy

$$\frac{p^2}{2m} + V = T + V$$

$$\therefore f = Ce^{-(i/\hbar)Et}$$

$$f = Ce^{-i\omega t} \qquad (5)$$

Where $E = h\nu$ and c is an arbitrary constant. This mean f depends on time harmonically with frequency ω. The normalised solⁿ of time dependent eqⁿ is

$$\psi(r,t) = \psi(r)e^{-i\omega t} = \psi(\vec{r})e^{-i/\hbar Et} \qquad (6)$$

The probability density $\psi^*\psi$ is now independent of time.

$$\therefore \psi^*\psi = \psi^*(\vec{r})e^{+i/\hbar \cdot Et}\,\psi(\vec{r})e^{-i/\hbar \cdot Et}$$

$$= \psi^*(\vec{r})\,\psi(\vec{r})$$

$=$ Independent of time.

This wave function 6 is said to represent a stationary state of the particle. ψ is said to be an eigenfunction of the operator H.